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By

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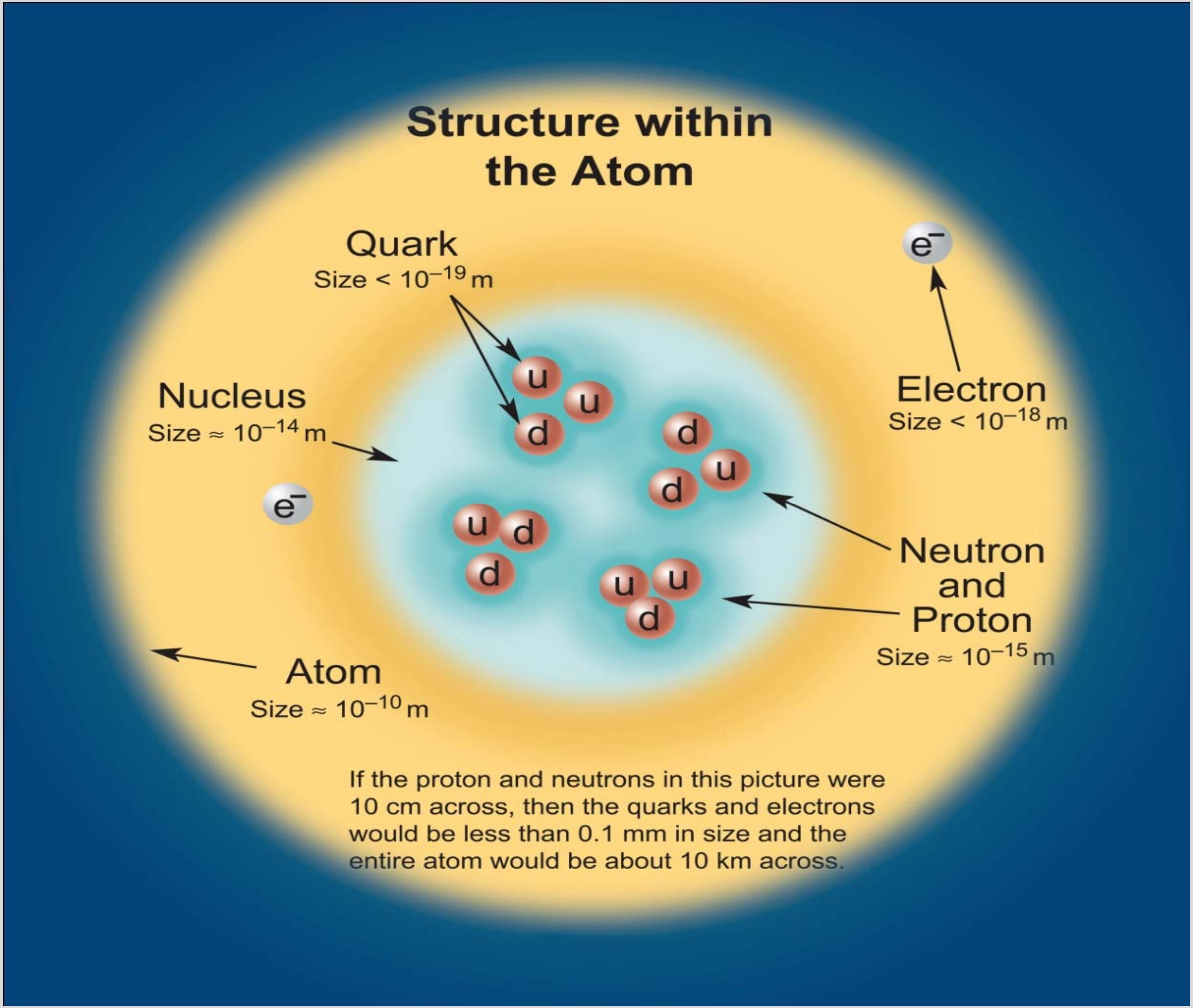
Structure within the Atom



Quark
Size < 10^{-19} m

Nucleus
Size ≈ 10^{-14} m

Electron
Size < 10^{-18} m

Neutron and Proton
Size ≈ 10^{-15} m

Atom
Size ≈ 10^{-10} m

If the proton and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

The Atomic Nucleus

- Glvfryhu|#ri#wkh#Qhxwurq
- Qxfohdu#Surshuwlhv
- Wkh#Ghxwhurq
- Qxfohdu#Irufhv
- Qxfohdu#Vwdelolw|
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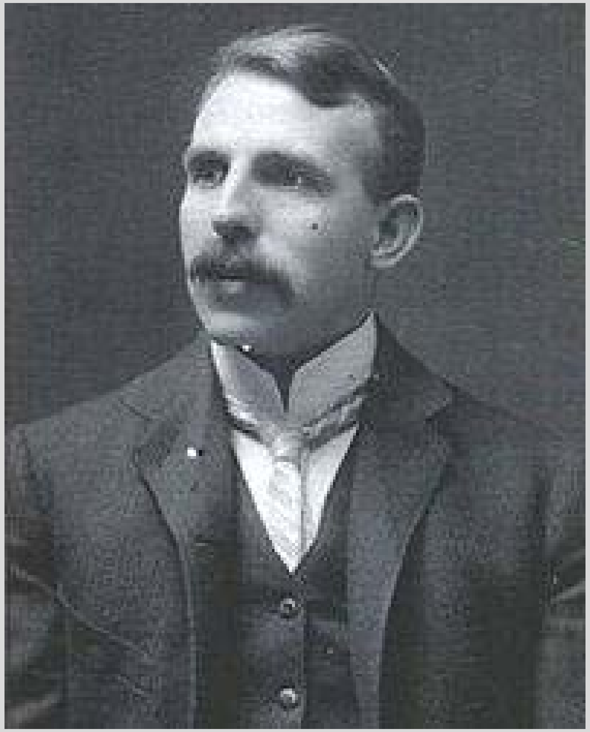
- Udglrdfwlyh#Qxfolghv#



Ernest Rutherford (1871-1937)

Discovery of the Neutron

Rutherford proposed the atomic structure with the massive nucleus in 1911. But only protons and electrons were known.

Reasons why electrons can't exist within the nucleus:

Nuclear size

The uncertainty principle puts a lower limit on its kinetic energy that is much larger that any kinetic energy observed for an electron emitted from nuclei.

Nuclear spin

If a deuteron consists of protons and electrons, the deuteron must contain 2 protons and 1 electron. A nucleus composed of 3 fermions must result in a half-integral spin. But it has been measured to be 1.

Nuclear magnetic moment

The measured nuclear magnetic moments are on the same order of magnitude as the proton's. The magnetic moment of an electron is over 1000 times larger than that of a proton.

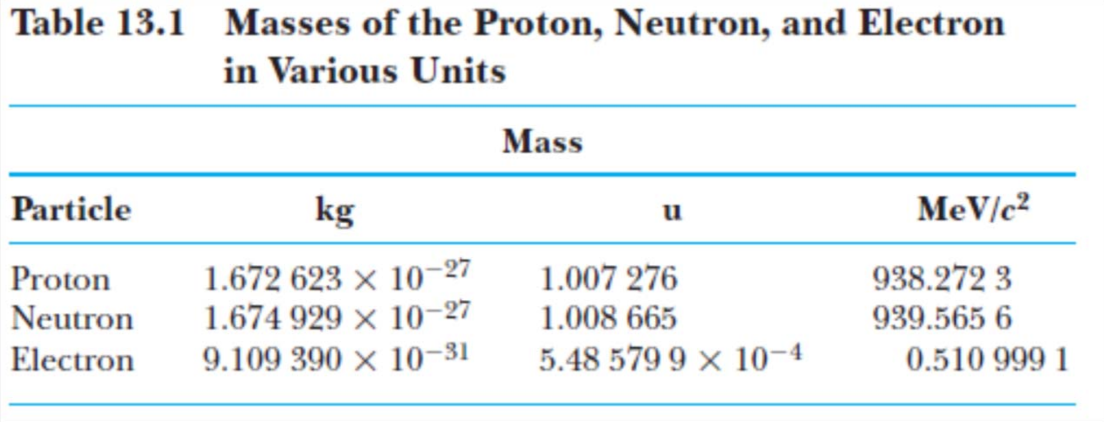
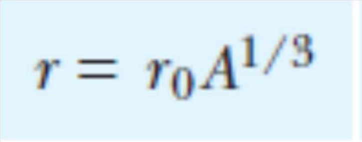


Table 13.1 Masses of the Proton, Neutron, and Electron in Various Units

	Mass		
Particle	**kg**	**u**	**MeV/c^2**
Proton	$1.672\,623 \times 10^{-27}$	1.007 276	938.272 3
Neutron	$1.674\,929 \times 10^{-27}$	1.008 665	939.565 6
Electron	$9.109\,390 \times 10^{-31}$	$5.48\,579\,9 \times 10^{-4}$	0.510 999 1



$$r = r_0 A^{1/3}$$

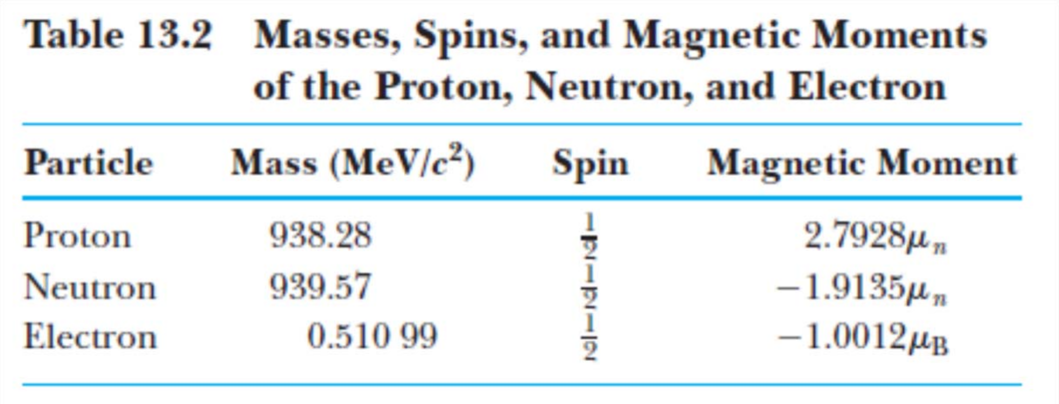


Table 13.2 Masses, Spins, and Magnetic Moments of the Proton, Neutron, and Electron

Particle	Mass (MeV/c^2)	Spin	Magnetic Moment
Proton	938.28	$\frac{1}{2}$	$2.7928\mu_n$
Neutron	939.57	$\frac{1}{2}$	$-1.9135\mu_n$
Electron	0.510 99	$\frac{1}{2}$	$-1.0012\mu_B$

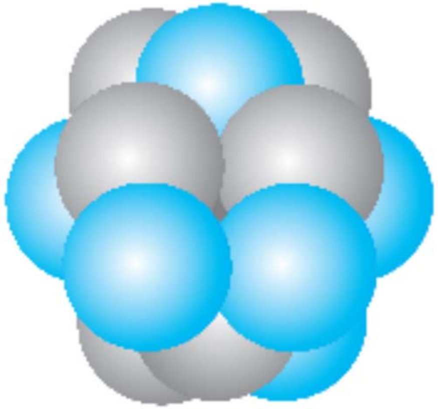
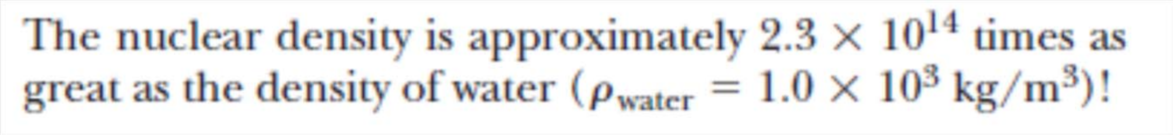


Figure 13.3 A nucleus can be modeled as a cluster of tightly packed spheres, each of which is a nucleon.



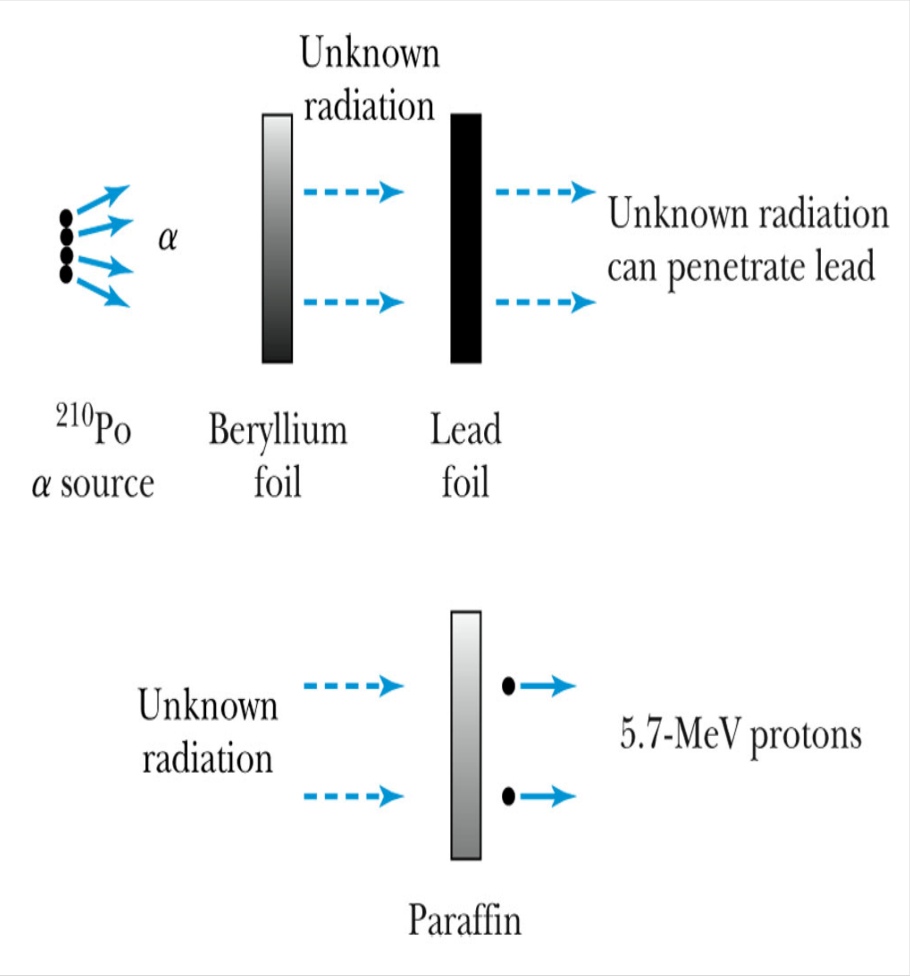
The nuclear density is approximately 2.3×10^{14} times as great as the density of water ($\rho_{\text{water}} = 1.0 \times 10^3\ \text{kg/m}^3$)!

Discovery of the Neutron

In 1930 German physicists Bothe and Becker used a radioactive polonium source that emitted α particles.

When these α particles bombarded beryllium, the radiation generated in Be (neutrons) was uncharged and penetrated several centimeters of lead.

James Chadwick confirmed the discovery in 1932.



By 1932, scientists realized that the nucleus was made up of protons and neutrons (both now called **nucleons**).

Nuclear Properties

The nuclear charge is $+e$ times the number (Z) of protons.

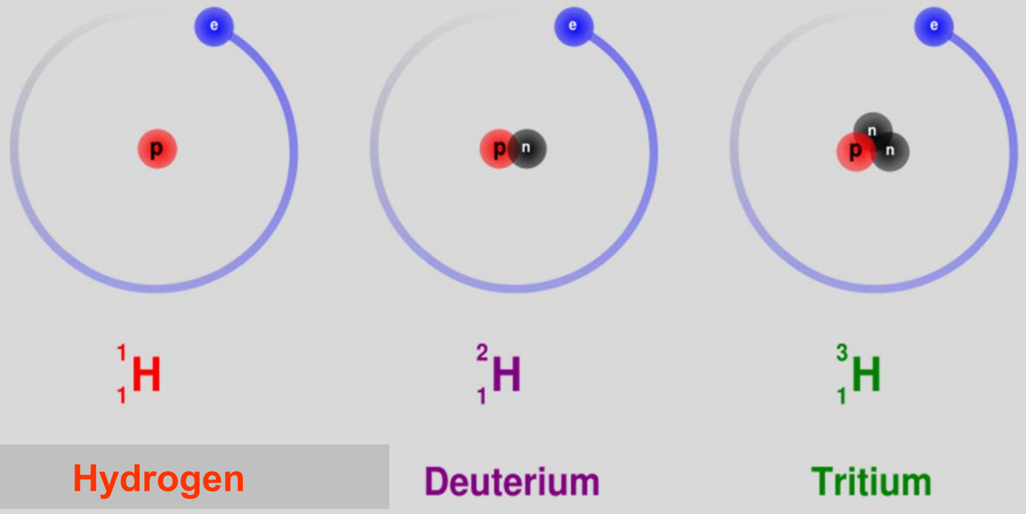
Hydrogen's **isotopes**:

Deuterium: Heavy hydrogen. Has a neutron as well as a proton in its nucleus.

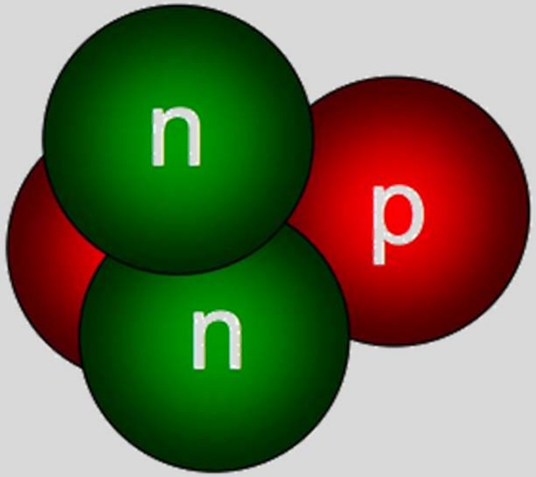
Tritium: Heavier hydrogen! Has two neutrons and one proton.

The nuclei of the deuterium and tritium atoms are called *deuterons* and *tritons*.

Atoms with the same Z, but different mass number A, are called **isotopes**.



Nuclear Properties



The symbol of an atomic nucleus

is $^{A}_{Z}X_{N}$.

$^{A}_{Z}X_{N}$ where Z = atomic number (number of protons)

N = neutron number (number of neutrons)

A = mass number ($Z+N$)

X = chemical element symbol

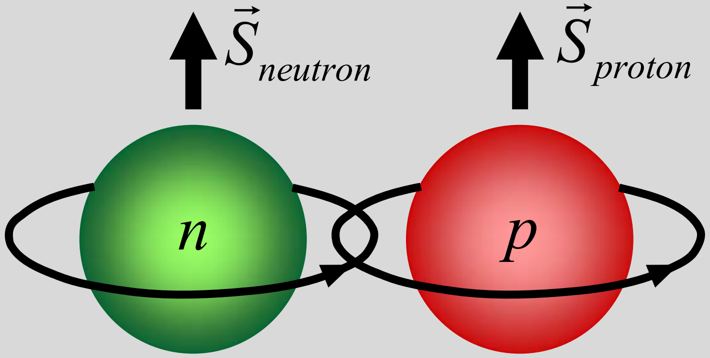
Each nuclear species with a given Z and A is called a **nuclide**.

Z characterizes a chemical element.

The chemical properties of X are nearly independent of N.

Atoms with the same Z, but different mass number A, are called **isotopes**.

Nuclear Force and Spin



Deuteron

The nuclear force is **spin dependent**.

The neutron and proton spins are aligned **parallel** for the bound state of the deuteron, and there is no bound state with the spins anti-aligned.

The *nn* system is more difficult to study because free neutrons are not stable.

The nuclear potential between two nucleons seems independent of their electric charge.

Nuclear Properties

Atomic masses are denoted by the atomic mass unit, with symbol: u.

1 u = 1.66054×10^{-27} kg = 931.49 MeV/c^2

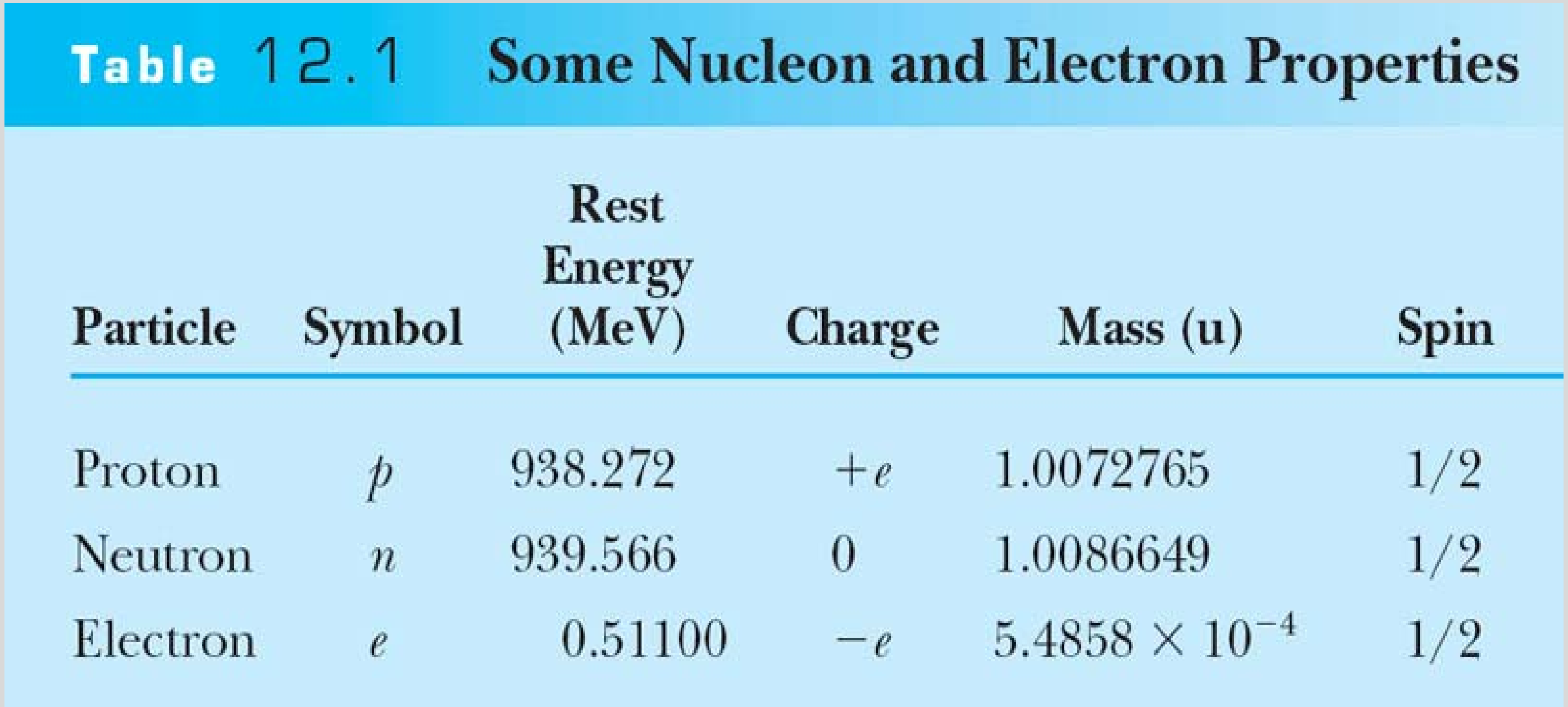


Table 12.1 Some Nucleon and Electron Properties

Particle	Symbol	Rest Energy (MeV)	Charge	Mass (u)	Spin
Proton	p	938.272	$+e$	1.0072765	1/2
Neutron	n	939.566	0	1.0086649	1/2
Electron	e	0.51100	$-e$	5.4858×10^{-4}	1/2

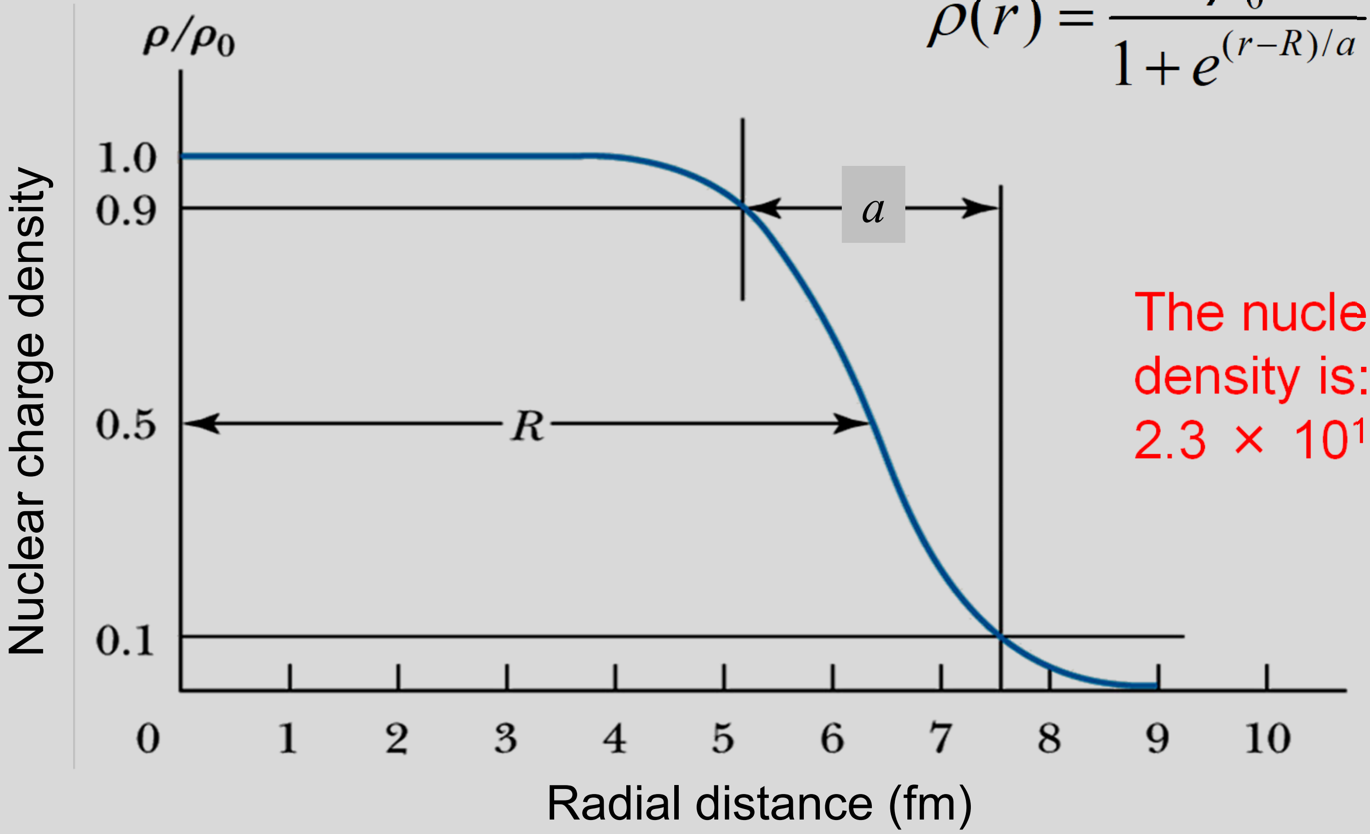
Sizes and Shapes of Nuclei

Since $V \propto R^3$, the nuclear radius is: $R = r_0 A^{1/3}$ where $r_0 \approx 1.2 \times 10^{-15}$ m.

We use the **femtometer** with 1 fm = 10^{-15} m, or the fermi.

The lightest nuclei are described by the Fermi distribution for the nuclear charge density $\rho(r)$:

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$



The nuclear mass density is: 2.3 × 10^{17} kg / m^3.

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- The proton’s intrinsic magnetic moment points in the same direction as its intrinsic spin angular momentum (as it is positive).
- Nuclear magnetic moments are measured in units of the nuclear magneton μ_N.

$$\mu_\mathrm{N} = \frac{e\hbar}{2m_p}$$

- The divisor in calculating μ_N is the proton mass m_p, which makes the nuclear magneton 1836 times smaller than the Bohr magneton.
- The proton magnetic moment is $\mu_p = 2.79\ \mu_\mathrm{N}$.
- The magnetic moment of the electron is $\mu_e = -1.00116\ \mu_\mathrm{B}$.
- The neutron magnetic moment is $\mu_n = -1.91\ \mu_\mathrm{N}$.
- The *nonzero* neutron magnetic moment implies that the neutron has negative and positive internal charge components at different radii.
- Complex internal *charge distribution*.

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$^{2}_{1}H$

$$^{A=Z+N}_{Z}Atomic\_symbol$$

The deuteron mass = 2.013553 u (mass of a proton + mass of a neutron minus the mass equivalent of the binding energy).

- The mass of a deuteron atom = 2.014102 u.
- The difference = 0.000549 u. → the mass of an electron and take off its binding energy mass equivalent 13.6 eV / c^2.
- The deuteron nucleus is bound $^{2}_{1}H$ by a mass-energy B_d.
- The mass of a deuteron is

$$m_d = m_p + m_n - B_d / c^2$$

- Add an electron mass to each side and ignore its binding energy

$$m_d + m_e = m_p + m_n + m_e - B_d / c^2$$

1 proton plus 1 neutron = 2.0159414 u = mass of the nucleus of deuterium ???

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- $m_d + m_e$ is the atomic deuterium mass $M(^2\mathrm{H})$ and $m_p + m_e$ is the atomic hydrogen mass.

$$M(^2\mathrm{H}) = m_n + M(^1\mathrm{H}) - B_d / c^2$$

- Because the electron masses cancel in almost all nuclear-mass difference calculations, we use atomic masses rather than nuclear masses.

$$m_n = 1.008665 \text{ u} \qquad \text{Neutron mass}$$

$$M(^1\mathrm{H}) = 1.007825 \text{ u} \qquad \text{Atomic hydrogen mass}$$

$$M(^2\mathrm{H}) = 2.014102 \text{ u} \qquad \text{Atomic deuterium mass}$$

$$B_d / c = m_n + M(^1\mathrm{H}) - M(^2\mathrm{H}) = 0.002388 \text{ u}$$

- Convert this to energy using u = 931.5 MeV / c^2.

$$B_d = 0.002388\ c^2 \cdot \mathrm{u}\left(\frac{931.5 \text{ MeV}}{c^2 \cdot \mathrm{u}}\right) = 2.224 \text{ MeV}$$

- Even for heavier nuclei we neglect the electron binding energies (much larger than 13.6 eV) because the nuclear binding energy (e.g. 2 MeV) is hundreds of thousands times greater.

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- The binding energy of any nucleus ${}^{A}_{Z}X$ = the energy required to separate the nucleus into free neutrons and protons.

$$B\left({}^{A}_{Z}X\right)=\left[Nm_n+ZM\left({}^{1}\mathrm{H}\right)-M\left({}^{A}_{Z}X\right)\right]c^2$$

Experimental Determination of Nuclear Binding Energies

- Check the 2.22-MeV binding energy by using a nuclear reaction. We scatter gamma rays from deuteron gas and look for the breakup of a deuteron into a neutron and a proton:

$$\gamma+d\rightarrow n+p$$

- This nuclear reaction is called *photodisintegration* or a *photonuclear reaction*.
- The mass-energy relation is

$$hf+M({}^{2}\mathrm{H})c^2=m_nc^2+M({}^{1}\mathrm{H})c^2+K_n+K_p$$

- where *hf* is the incident photon energy.

K_n and K_p are the neutron and proton kinetic energies.

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- The minimum energy required for the photodisintegration:
- Momentum must be conserved in the reaction (K_n, $K_p \neq 0$).

$$hf_{\min} = B_d \left[1 + \frac{B_d}{2M({}^2\mathrm{H})c^2} \right]$$

- Experiment shows that a photon of energy less than 2.22 MeV cannot dissociate a deuteron.

- **Deuteron Spin and Magnetic Moment**
- Deuteron’s nuclear spin quantum number is 1. This indicates the neutron and proton spins are aligned parallel to each other.
- The nuclear magnetic moment of a deuteron is 0.86 μ_N ≈ the sum of the free proton and neutron 2.79 μ_N − 1.91 μ_N = 0.88μ_N.

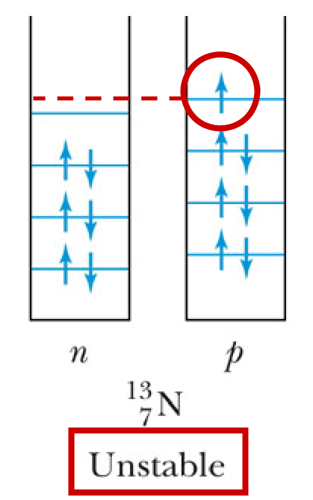
Nuclear Models

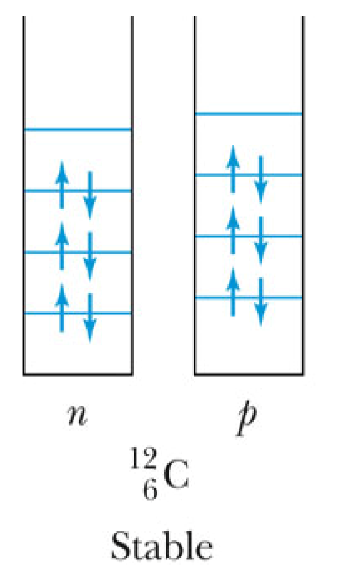
Most stable nuclides have both even Z and even N (**even-even nuclides**).

Nucleons obey the Pauli exclusion principle. So opposite spins can occupy the same states and so are more stable.

Energy-level diagrams for ^{12}C, ^{13}N, and ^{13}C.

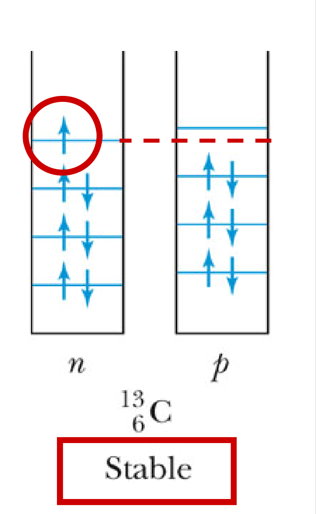
If we add one proton to ^{12}C to make ^{13}N





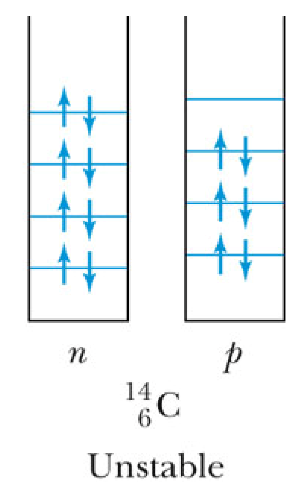
^{12}C is stable because it's even-even.

If we add one neutron to ^{12}C to make ^{13}C:



Nuclear Models

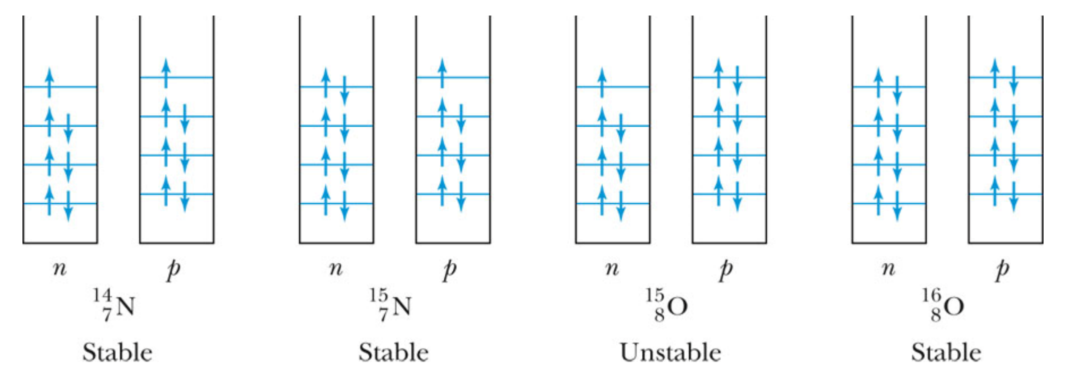
When we add another neutron to produce ^{14}C, we find it's just barely unstable.



For small nuclei, nature prefers the number of neutrons and protons to be about equal, so ^{14}C isn't stable.

But nature hates $Z > N$! This helps explain why ^{13}C is stable, but not ^{13}N, which has $Z = 7$ and $N = 6$.

More examples of stability and instability.

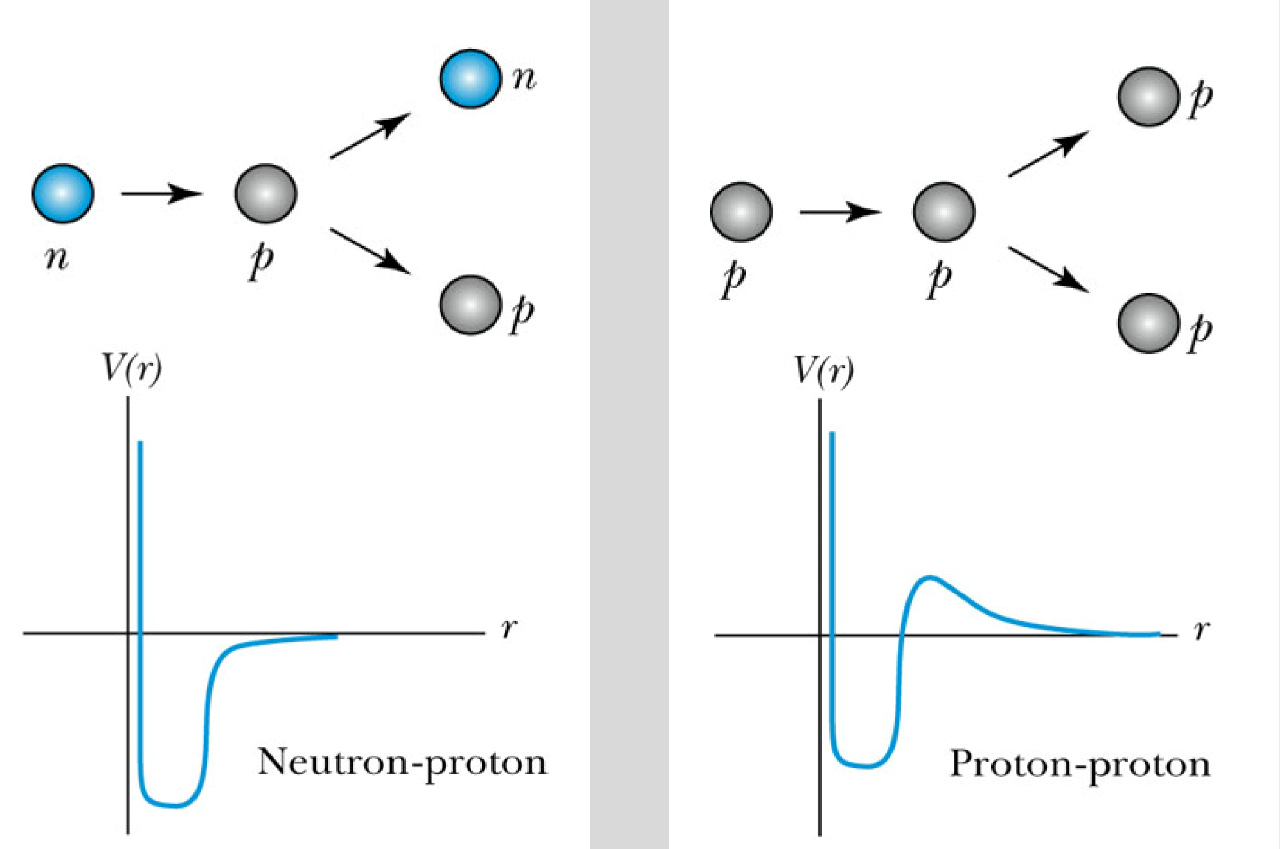


Nuclear Forces

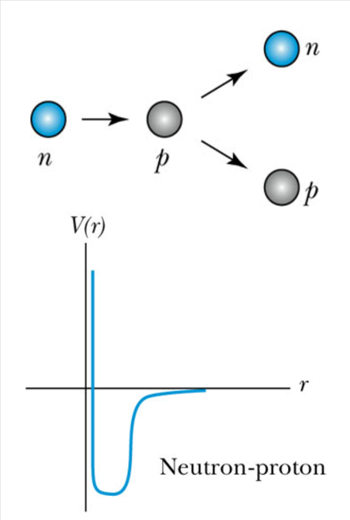
Because there’s no negative charge in the nucleus, it’s clear that a new force is involved. The nuclear force is called the strong force for the obvious reason!

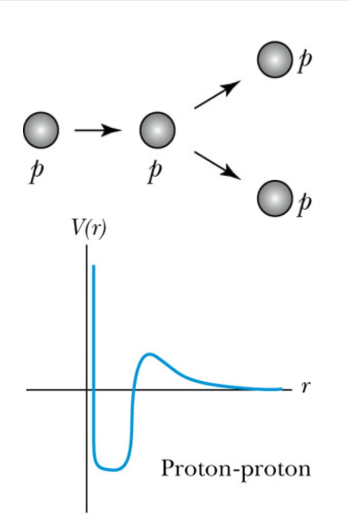
The nuclear potential energy vs. distance

The angular distribution of nucleons scattered by other nucleons tells us the nuclear potential.



Nuclear Forces





The proton has charge at radii up to 1 fm.

The inter-nucleon potential has a “hard core” that prevents the nucleons from approaching each other closer than about 0.4 fm.

The nuclear force has a very *short range*. Two nucleons within about 2 fm of each other feel an attractive force. But it falls to zero abruptly for separations greater than a few fm.

The only difference between the *np* and *pp* potentials is the Coulomb potential shown for $r \geq 3$ fm for the *pp* force.

Interior nucleons are completely surrounded by other nucleons, with which they interact.

Nuclear Stability

- The line representing the stable nuclides is the **line of stability**.
- It appears that for $A \le 40$, nature prefers the number of protons and neutrons in the nucleus to be about the same $Z \approx N$.
- However, for $A \ge 40$, there is a decided preference for $N > Z$ because the nuclear force is independent of whether the particles are *nn*, *np*, or *pp*.

- As the number of protons increases, the Coulomb force between all the protons becomes stronger until it eventually affects the binding significantly.

- The work required to bring the charge inside the sphere from infinity is

$$\Delta E_{\text{Coul}} = \frac{3}{5}\frac{(Ze)^2}{4\pi\varepsilon_0 R}$$

Nuclear Stability

- For a single proton,

$$\Delta E_{\text{Coul}} = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R}$$

- The total Coulomb repulsion energy in a nucleus is

$$\Delta E_{\text{Coul}} = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

- For heavy nuclei, the nucleus will have a preference for fewer protons than neutrons because of the large Coulomb repulsion energy.

- Most stable nuclides have both even *Z* and even *N* (even-even nuclides).
- Only four stable nuclides have odd *Z* and odd *N* (odd-odd nuclides).

$^{2}_{1}\mathrm{H}$, $^{6}_{3}\mathrm{Li}$, $^{10}_{5}\mathrm{B}$, and $^{14}_{7}\mathrm{N}$.

The Liquid Drop Model

- Treats the nucleus as a collection of interacting particles in a liquid drop.
- The total binding energy, the semi-empirical mass formula is

$$B\left({}^{A}_{Z}X\right) = \underbrace{a_V A}_{\text{Volume Energy}} - \underbrace{a_A A^{2/3}}_{\text{Surface Energy}} - \underbrace{\frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 r}}_{\text{Coulomb Energy}} - a_S \frac{(N-Z)^2}{A} + \delta$$

- The volume term (a_v) indicates that the binding energy is approximately the sum of all the interactions between the nucleons.
- The second term is called the *surface effect* because the nucleons on the nuclear surface are not completely surrounded by other nucleons.
- The third term is the Coulomb energy in Equation.

The Liquid Drop Model

- The fourth term is due to the symmetry energy. In the absence of Coulomb forces, the nucleus prefers to have $N \approx Z$ and has a quantum-mechanical origin, depending on the exclusion principle.
- The last term is due to the pairing energy and reflects the fact that the nucleus is more stable for even-even nuclides. Use values given by Fermi to determine this term.

$$\begin{aligned} a_V &= 14\ \text{MeV} && \text{Volume} \\ a_A &= 13\ \text{MeV} && \text{Surface} \\ a_S &= 19\ \text{MeV} && \text{Symmetry} \end{aligned}$$

$$\text{Pairing } \delta = \begin{cases} +\Delta & \text{for even-even nuclei} \\ 0 & \text{for odd-}A\text{ (even-odd, odd-even) nuclei} \\ -\Delta & \text{for odd-odd nuclei} \end{cases}$$

- where $\Delta = 33\ \text{MeV} \cdot A^{-3/4}$
- No nuclide heavier than ${}^{238}_{92}\mathrm{U}$ has been found in nature. If they ever existed, they must have decayed so quickly that quantities sufficient to measure no longer exist.

Nuclear Stability

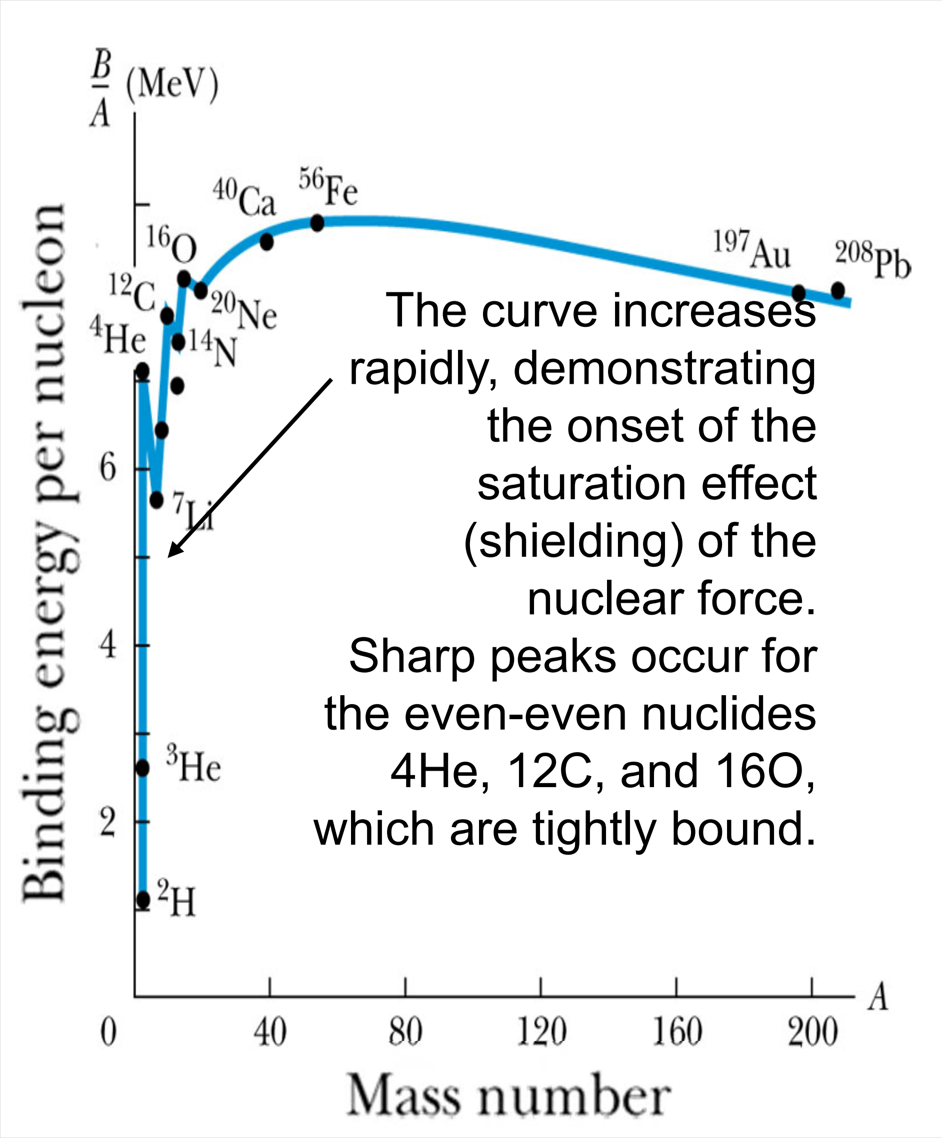
The **binding energy** of a nucleus against dissociation into any other possible combination of nucleons.

Example: nucleus X, which could decay to other nuclei R and S:

$$B = \left[M(R) + M(S) - M({}^{A}_{Z}X) \right] c^2$$

where M indicates the mass.

If the binding energy is low or negative, the nucleus will decay.



The curve increases rapidly, demonstrating the onset of the saturation effect (shielding) of the nuclear force.

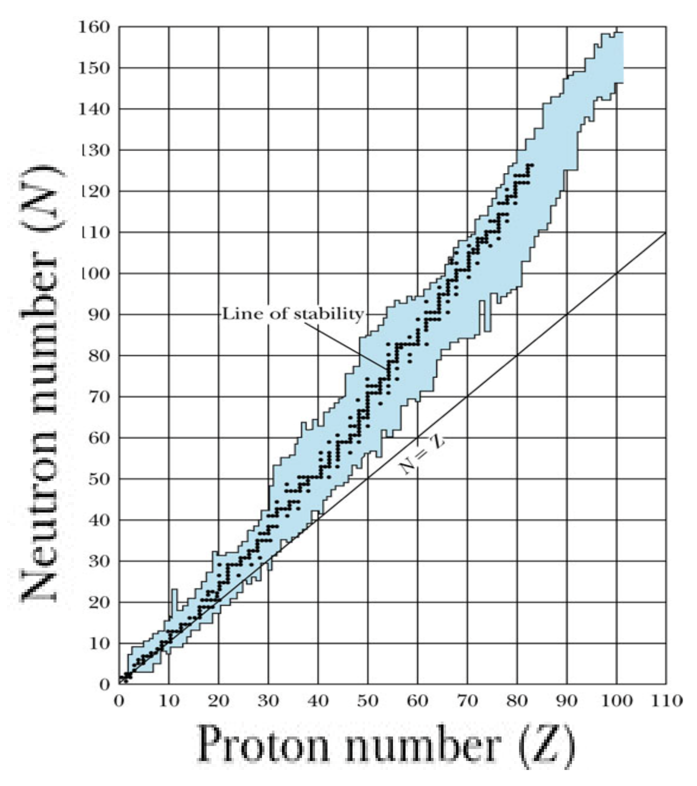
Sharp peaks occur for the even-even nuclides 4He, 12C, and 16O, which are tightly bound.

Nuclear Stability

The line representing the stable nuclides is the **line of stability**.

For $A \leq 40$, nature prefers the number of protons and neutrons in the nucleus to be about the same $Z \approx N$.

But, for $A \geq 40$, there is a preference for $N > Z$.



But the nuclear force is
independent of which particles (*nn*, *np*, or *pp*). So why is this?

Answer: As the number of protons increases, the Coulomb force between all the protons becomes stronger until it eventually affects the binding significantly.

Radioactive Decay

Marie Curie and her husband Pierre discovered polonium and radium in 1898.

The simplest decay form is that of a gamma ray, which represents the nucleus changing from an excited state to lower energy state.

Other modes of decay include emission of α particles, β particles, protons, neutrons, and fission (a large nucleus breaking into two intermediate-size nuclei).

The disintegrations or decays per unit time (**activity**).

$$Activity = -\frac{dN}{dt} = R$$

where dN/dt is negative because total number N decreases with time.

Radioactive Decay

- SI unit of activity is the becquerel: 1 Bq = 1 decay / s
- Recent use is the Curie (Ci) 3.7×10^{10} decays / s

- If N(t) is the number of radioactive nuclei in a sample at time *t*, and λ (**decay constant**) is the probability per unit time that any given nucleus will decay:

$$
\begin{aligned}
&R = \lambda N(t) \\
&dN(t) = -R\,dt = -\lambda N(t)\,dt \\
&\int \frac{dN}{N} = -\int \lambda\,dt \\
&\ln N = -\lambda t + \text{constant} \\
&N(t) = e^{-\lambda t + \text{constant}}
\end{aligned}
$$

- If we let $N(t = 0) \equiv N_0$

$N(t) = N_0 e^{-\lambda t}$ **----- radioactive decay law**

Radioactive Decay

- The activity R is

$$R = \lambda N(t) = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

  where R_0 is the initial activity at $t = 0$
- It is common to refer to the half-life $t_{1/2}$ or the mean lifetime τ rather than its decay constant.

$$N(t_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

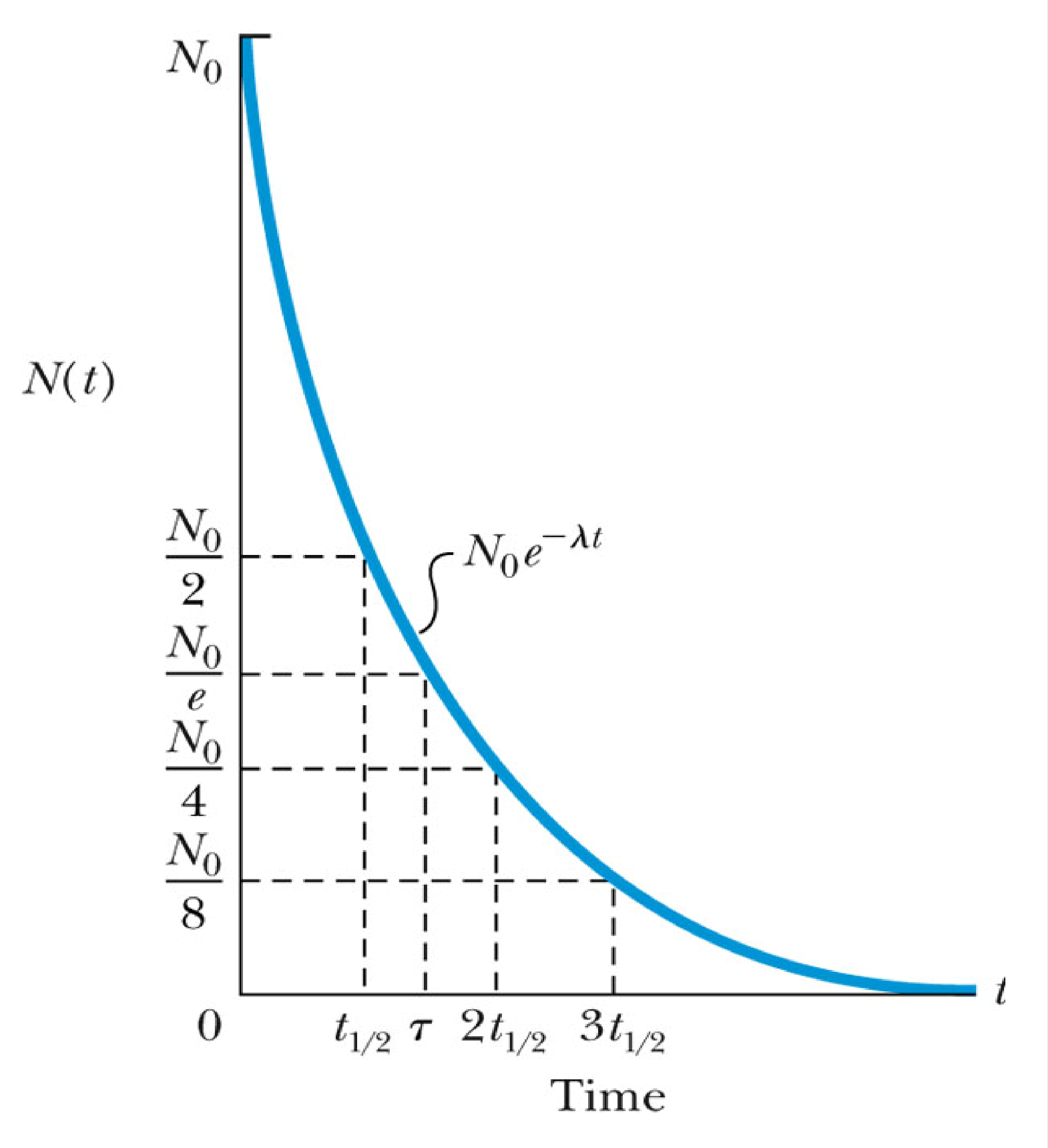
$$\ln\left(\frac{1}{2}\right) = \ln(e^{-\lambda t_{1/2}}) = -\lambda t_{1/2}$$

- The half-life is $t_{1/2} = \frac{-\ln(1/2)}{\lambda} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}$
- The mean lifetime is $\tau = \frac{1}{\lambda} = \frac{t_{1/2}}{\ln(2)}$

Radioactive Decay

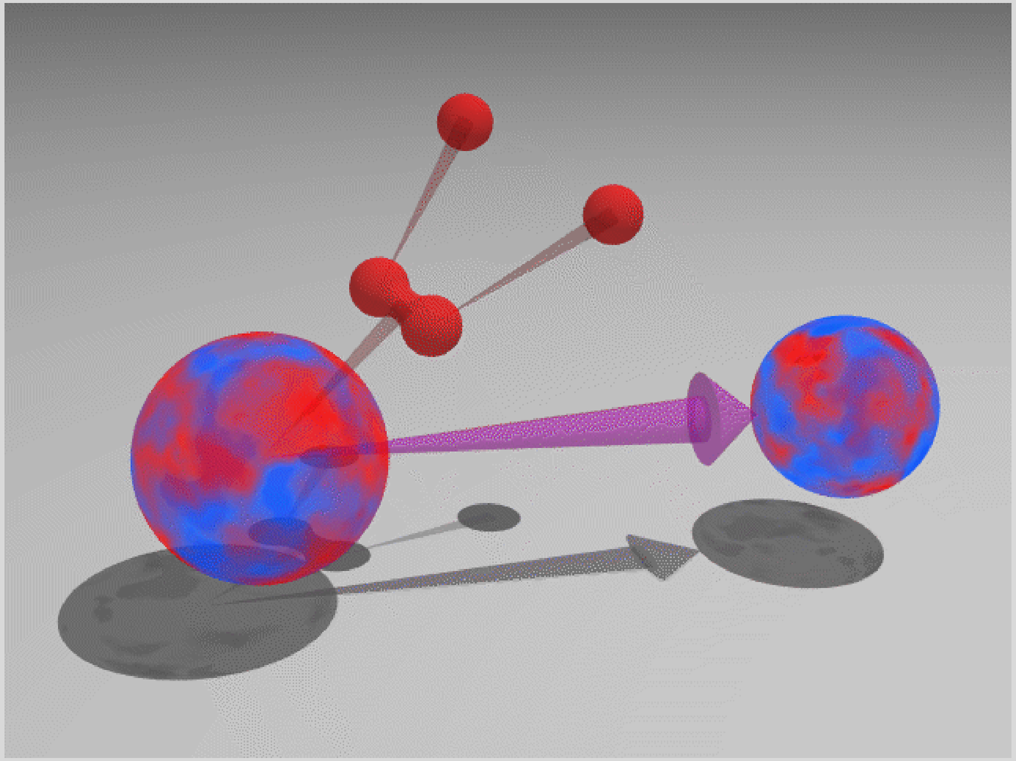
The number of radioactive nuclei as a function of time.

The time for the number of nuclei to drop to one half its original value is the well-known **half life**.



Alpha, Beta, and Gamma Decay

When a nucleus decays, all the conservation laws must be observed:



Mass-energy

Linear momentum

Angular momentum

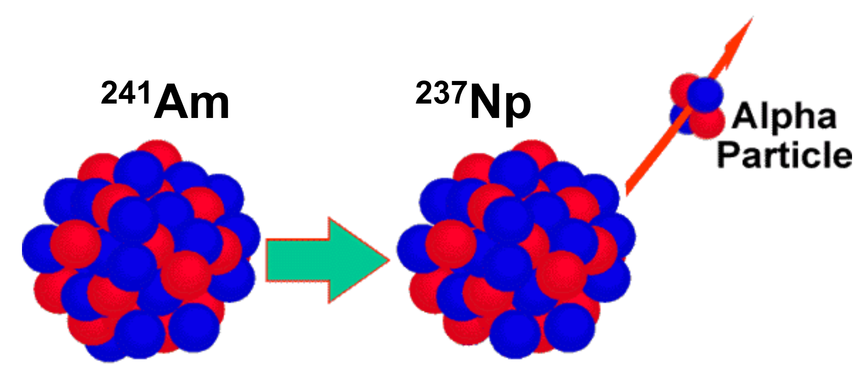
Electric charge

Conservation of nucleons

The total number of nucleons (A, the mass number) must also be conserved in a low-energy nuclear reaction or decay.

Alpha Decay

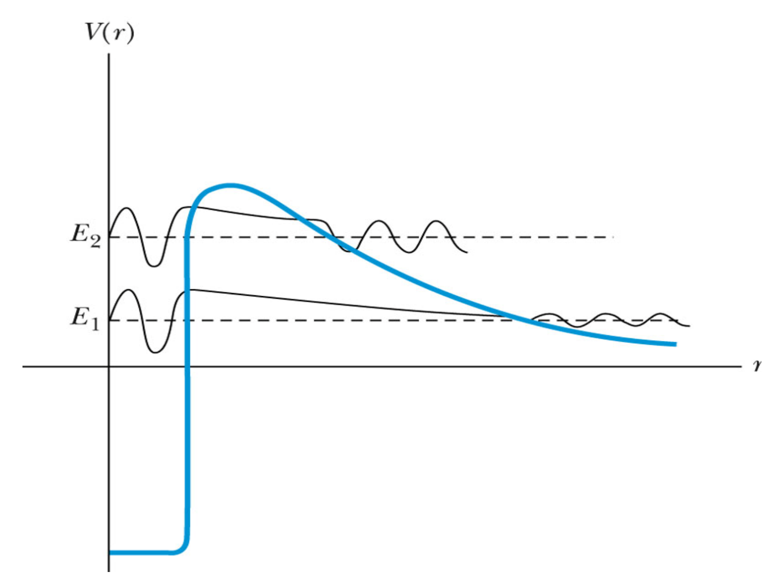
The barrier for alpha decay is > 20 MeV, but the kinetic energies of alpha particles emitted from nuclei range from 4 to 10 MeV.



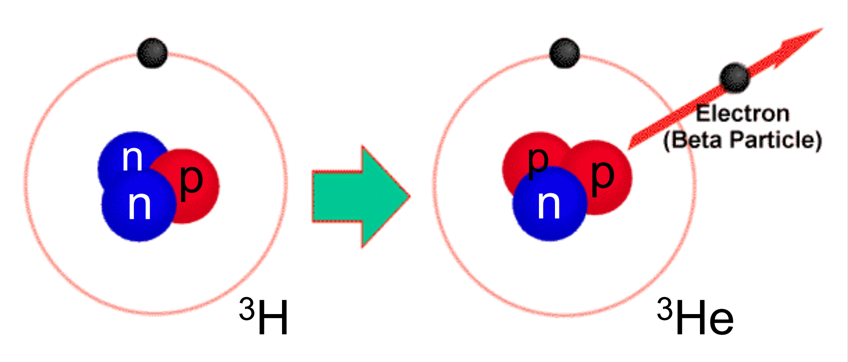
So it's impossible classically for the alpha particle to penetrate the barrier, but they can quantum-mechanically tunnel through it.

A higher energy E_2 has much higher probability than does a lower energy E_1.

There is a correlation between lower energies and greater difficulty of escaping (longer lifetimes).



Beta decay



Unstable nuclei may move closer to the line of stability by undergoing **beta decay**.

The decay of a free neutron is: $n \rightarrow p + \beta^-$

But wait! How can this happen? All three particles have spin ½! This reaction doesn't conserve angular momentum!

In 1930, Wolfgang Pauli suggested a new particle, the **neutrino,** ν, which must be produced in beta decay. It has spin quantum number ½ and charge 0, and carries away the additional missing spin and energy:

$$n \rightarrow p + \beta^- + \overline{\nu}$$

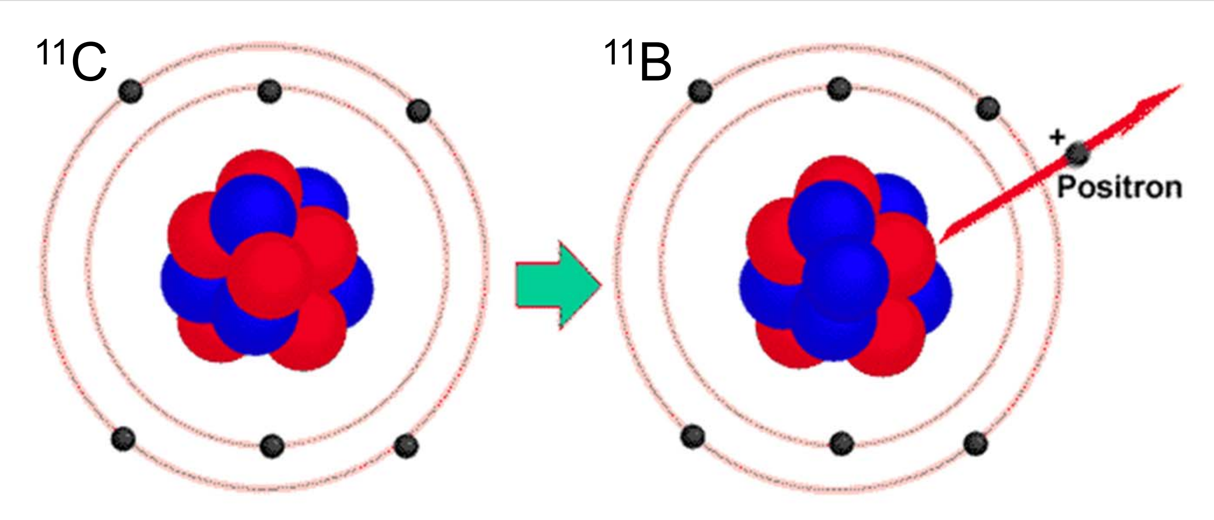
It's actually an anti-neutrino here.

The beta decay of ^{14}C to form ^{14}N, a stable nucleus, is:

$$^{14}\mathrm{C} \rightarrow {}^{14}\mathrm{N} + \beta^- + \overline{\nu} \qquad \beta^-\,\mathrm{decay}$$

Beta$^+$ Decay

Positron emission (β+ decay) occurs when the neutron to proton ratio is too small. A proton turns into a neutron, emitting a positron.

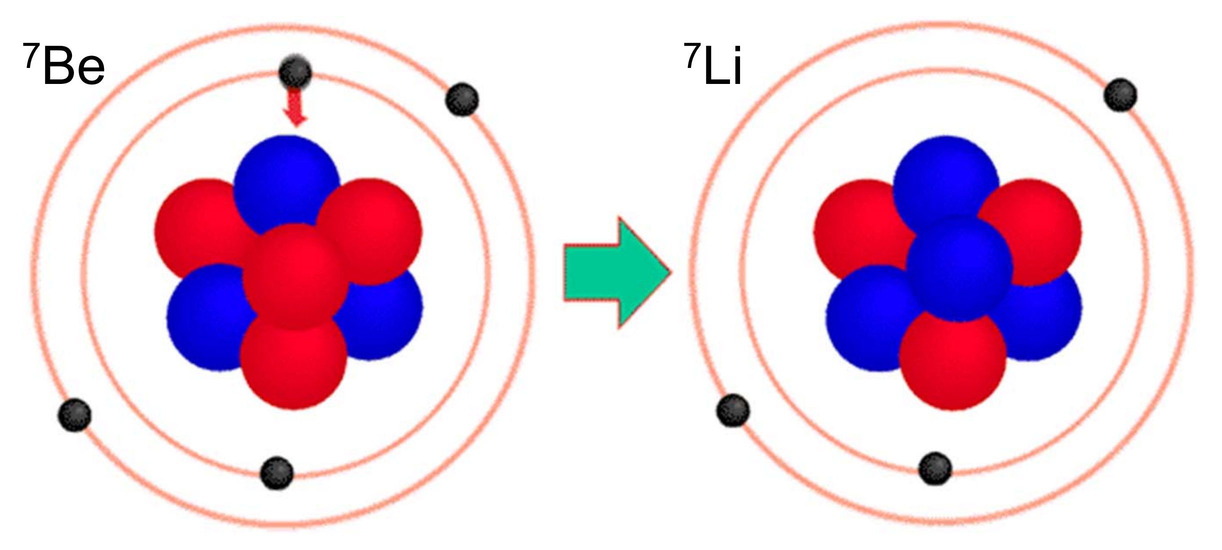


β^+ decay of ^{11}C to ^{11}B, emitting a positron and a neutrino:

$$^{11}\mathrm{C} \longrightarrow {}^{11}\mathrm{B} + e^+ + \nu_e$$

Electron Capture

Electron capture also occurs when the neutron to proton ratio is too small. The nucleus captures an electron, which turns a proton into a neutron.



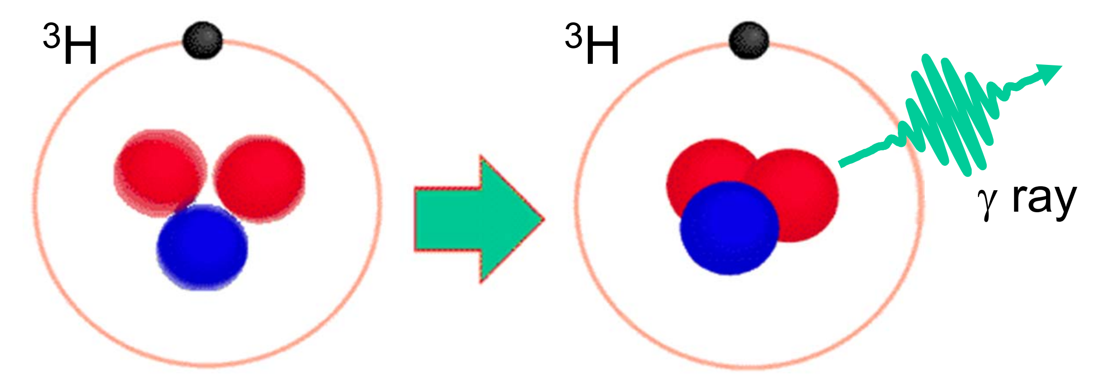
One of the inner orbital electrons is captured by a proton in the nucleus, forming a neutron and a neutrino:

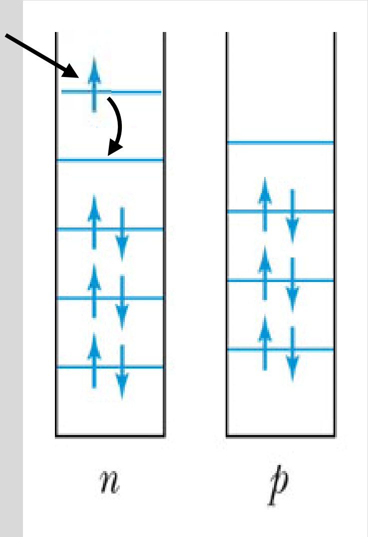
$$p + e- \;\rightarrow\; n + \nu_e$$

Gamma ray emission

Emission of photons happens from nuclei just as it does from atoms. Just as excited electrons decay to lower levels, nucleons do also, but with much higher energies (MeV!).

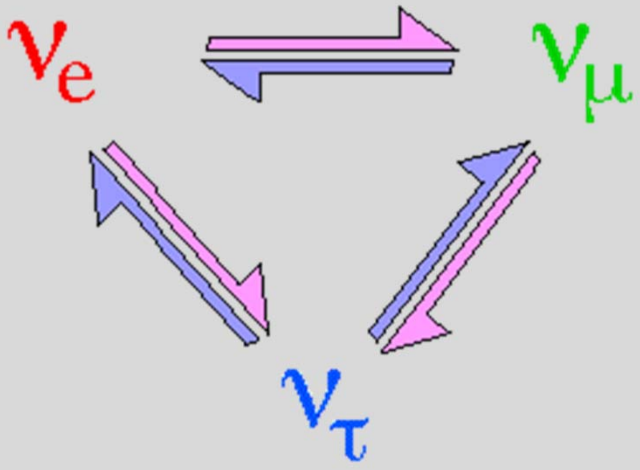
Gamma decay occurs when the nucleus is in an excited state and decays to a lower level.





Example: ^{60}Co decays to excited ^{60}Ni by beta decay, with the ^{60}Ni in an excited state. Then ^{60}Ni decays to the ground state by emitting two gamma rays in succession (1.17 MeV then 1.33 MeV).

Neutrinos



There are actually three
neutrinos and their anti-neutrinos.

All six have very little mass
(precisely how much we don’t know).

Neutrinos have no charge and do not interact electromagnetically.

They aren’t affected by the strong force of the nucleus.

They interact via the *weak* interaction.

Currently, experiments in deep mines filled with C_2Cl_4 are detecting them.

$$\nu_e + {}^{37}\mathrm{Cl} \rightarrow \beta^- + {}^{37}\mathrm{Ar}$$

Radioactive Nuclides

Some unstable nuclei found in nature exhibit natural radioactivity.

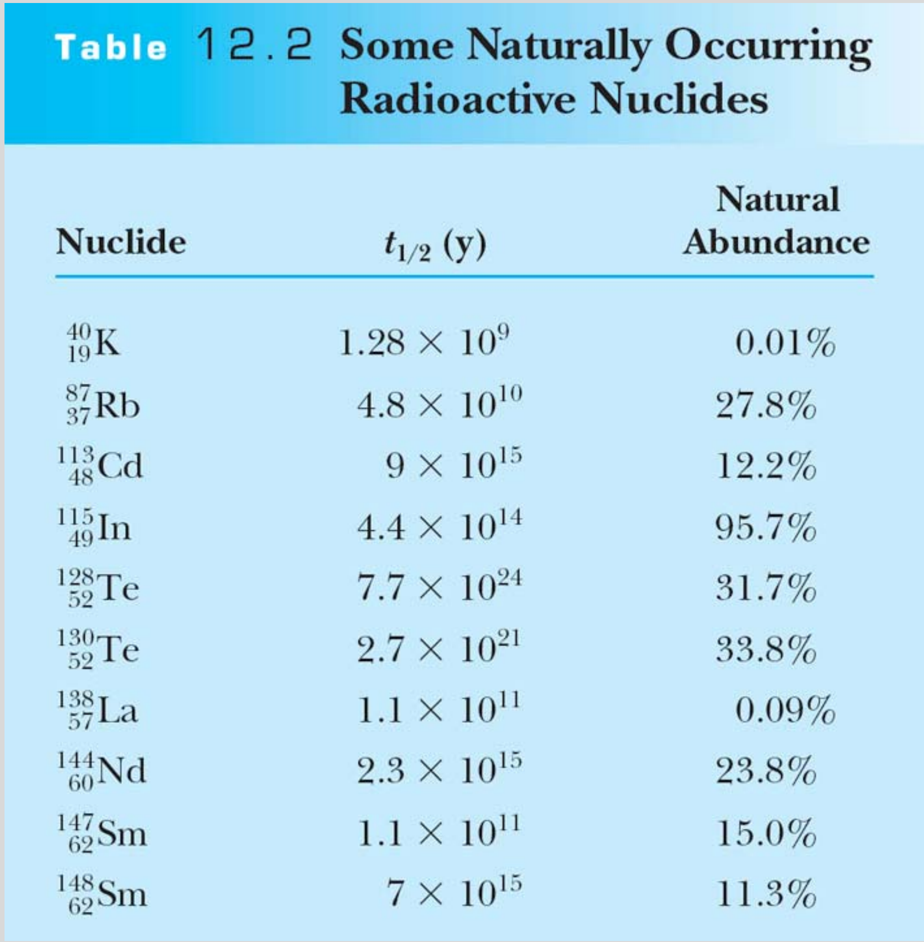


Table 12.2 Some Naturally Occurring Radioactive Nuclides

Nuclide	$t_{1/2}$ (y)	Natural Abundance
${}^{40}_{19}K$	1.28×10^{9}	0.01%
${}^{87}_{37}Rb$	4.8×10^{10}	27.8%
${}^{113}_{48}Cd$	9×10^{15}	12.2%
${}^{115}_{49}In$	4.4×10^{14}	95.7%
${}^{128}_{52}Te$	7.7×10^{24}	31.7%
${}^{130}_{52}Te$	2.7×10^{21}	33.8%
${}^{138}_{57}La$	1.1×10^{11}	0.09%
${}^{144}_{60}Nd$	2.3×10^{15}	23.8%
${}^{147}_{62}Sm$	1.1×10^{11}	15.0%
${}^{148}_{62}Sm$	7×10^{15}	11.3%

Other unstable nuclei (with much shorter lives) must be created in the lab.

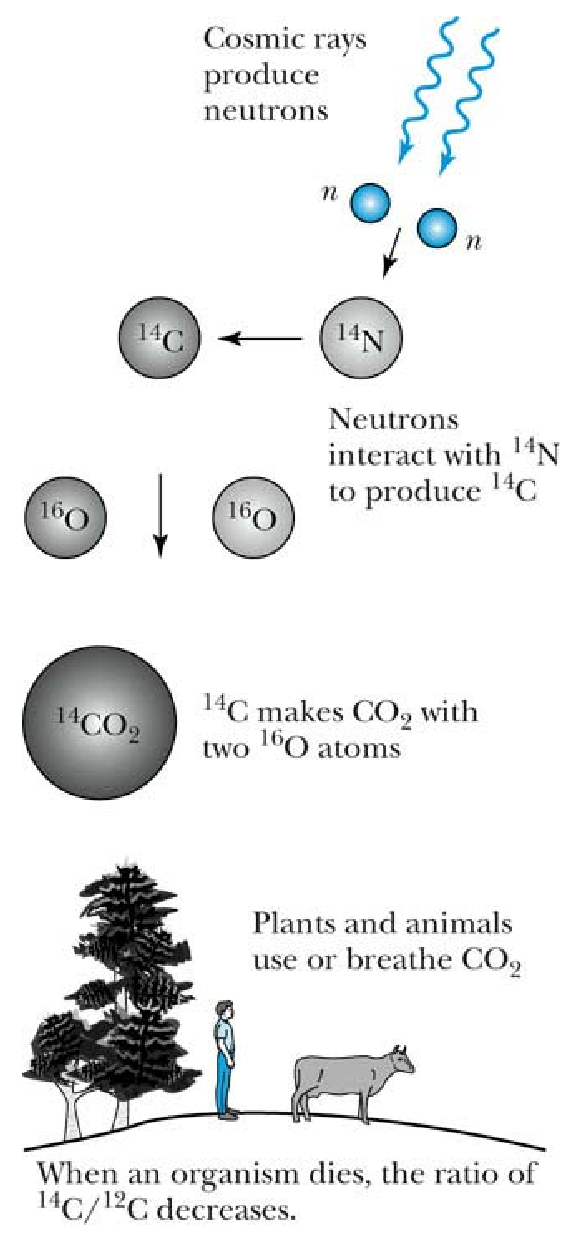
Radioactive Carbon Dating

Radioactive ^{14}C is produced in our atmosphere by the bombardment of ^{14}N by neutrons produced by cosmic rays.

$$n + {}^{14}\mathrm{N} \rightarrow {}^{14}\mathrm{C} + p$$

When living organisms die, their intake of ^{14}C ceases, and the ratio of ^{14}C / ^{12}C decreases as ^{14}C decays.

Because the half-life of ^{14}C is 5730 years, we can use the ^{14}C / ^{12}C ratio to determine the age of objects up to 45,000 old.



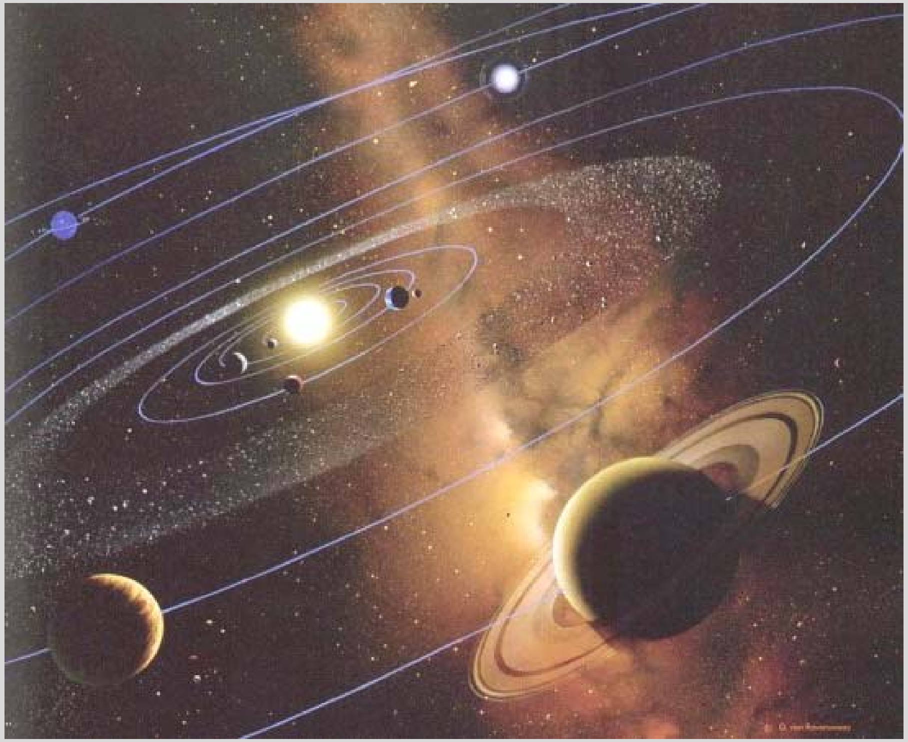
Potassium-argon dating

Potassium–argon dating is used in archeology. It’s based on the radioactive decay (positron emission) of an isotope of potassium ^{40}K (half-life = 1.248 × 10^9 yr) into argon (Ar). ^{40}Ar is able to escape liquid (molten) rock, but starts to accumulate when the rock solidifies (re-crystallizes).

Time since re-crystallization is determined by measuring the ratio of the amount of ^{40}Ar accumulated to the amount of ^{40}K remaining.

The long half-life of ^{40}K allows the method to be used to calculate the absolute age of samples older than a few thousand years.

Long-Time Dating Using Lead Isotopes



Nothing decays to or from ^{204}Pb.

A plot of the abundance ratio of ^{206}Pb / ^{204}Pb versus ^{207}Pb / ^{204}Pb can be a sensitive indicator of the age of lead ores.

Such techniques have been used to show that moon rocks and meteorites, believed to be left over from the formation of the solar system, are 4.55 billion years old.